Cybernetics & Human Knowing, Vol.7, no.4, 2000, pp. 91–95

Virtual Logic — Formal Arithmetic

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I. Introduction

This is column number 12.

Lets stop and look at that number twelve. A mere dozen, we use it all the time. And twelve is equal to the product of two twos and a three.

12= 2 x 2 x 3.

We were well programmed long ago to do this arithmetic, and yet it is quite the most mysterious subject.

A positive integer is said to be a *prime* if it is not equal to one, and if it has no integer factors other than itself and one. Here we see that 12 is factored into the product of the prime numbers 2 and 3. Factorization into primes is unique (a nice matter to prove and one of the first basic results in the theory of numbers). And there are infinitely many prime numbers! This last fact is something you can test by starting to list them

2,3,5,7,11,13,17,23,29,31,37,41,43,47,53,57,...

The proof that there are infinitely many prime numbers is due to Euclid. Just suppose that you have a finite list of primes. Then form their product and add one! This new number N is not divisible by any prime on your list. Hence either N is a new prime number, or it has a new prime factor. In either case, this simple remark shows that there cannot be only finitely many prime numbers, since any such list leads to the construction of new primes.

The purpose of this column is to go underneath the scene of numbers as we know them and to look at how these operations of addition and multiplication can be built in terms of a little technology of distinctions and the void. This is a story as old as creation herself, and we shall take some time to point out a mythological connection or two as we go along.

II. In the Beginning

In the beginning there was Everything/Nothing, a world with no distinctions, the Void. Of course, there was not even a world at this stage, and we do not really know how to describe how observers with understanding could arise from a world in which there really was nothing and no way to begin.

So the idea in exploring the possibility of an infinitely creative Void prior to the creation of All Things is to look at structures that we know in the world that we seem to know, and follow them back into simplicity. Some say — to deconstruct them, and this is a good term if it is understood rightly as saying that we might find out how to construct them in the process of deconstruction! In any case, I say lets follow a structure back into simplicity.

This will not be a linear process. Once we follow a structure back into what seems to be its essential simplicity, there is a new and wider view available, and this view compounded with what we already knew, leads to a new way to hold the entire matter and more possibility to move into even deeper simplicity.

It is a paradox. By moving into simplicity, we make room for a world with even greater complexity. And this complex world allows the movement into even greater simplicity. There is an infinite depth to simplicity, just as there is an infinite possibility for complexity.

III. One

One?

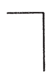
The Void of Everything/Nothing is certainly One. “It” (and by referring to it I naturally move away from it, for it is not an it. The Void, when named, is not the Void. There is no way to define, name, delineate or otherwise contain the uncontainable.

This very uncontainability makes the Void a One, since it certainly is not a Many. So we can certainly say with certainty that the Void is One. And at the same time there is no way to actually name the Void and so we might imagine that she has a secret and unpronouncable name. Void is not really her name. Void is a finger pointing to the moon.

You see that Mathematics, at this stage, is delicate. You can do mathematics in the neighborhood of the Void, but you had better be very careful to understand that reference just does not work in the everyday —up here in the trees—world full of things way. No. We have proved that the Void is One, because it certainly is not Many. But we have to take this very carefully, because, if we were to enter into the Void there would not even be One or None or Many.

On the other side, coming from our home in the trees, it is quite tempting to just say. Well , One is just one distinction. Like this:

Or perhaps this:



How can we reconcile that grand One of the One Void and the small one of one distinction? Clearly the answer lies in understanding that the one of the distinction stands for the form of distinction itself and that it is this form of distinction that we refer to when we distinguish the One. The one void is not an expression of a something, but rather an indication of our intuition of the form of distinction itself. Nothing is more distinguished than the Void, and so all aspects of distinction belong to it from the outside, as it were. In and of itself the void knows nothing, distinguishes nothing, is nothing.

Yet the void is Everything/Nothing and so all this, all this discussion is occurring in Void. This discussion pretends to make distinctions and to talk about the One and the Many. But it is fiction. It is all empty, and the only meaning that can possibly adhere to this discussion is emptiness.

For these reasons, we choose the mark

⏋

as the quintessential representative of one. The mark is seen to make a distinction in the plane on which it is drawn, and yet (being an abbreviated square) it provides an open pathway from inside to outside. The one mark unifies the sides that it divides.

IV. Many

We can proceed into arithmetic with

0 =
1 = ⏋
2 = ⏋⏋
3 = ⏋⏋⏋

and so on.

Addition is the juxtaposition of forms: a + b = ab.

Thus

1 + 1 = ⏋ + ⏋ = ⏋⏋ = 2.

Multiplication is more complex.

When we multiply 2 x 3 we either take two threes and add them together, or we take 3 twos and add these together. In either case we make an operator out of one number and use this operator to reproduce copies of the other number. We seek a way to put these patterns into our formalism.

Let n⏋ denote the operator corresponding to the number n. Here is how this will work. If we put n⏋ next to any operator m⏋ then

n⏋ m⏋

will create n copies of the number m (or m copies of the number n) and place them under the roof of a mark forming a new operator. Thus

$$\overline{5}\rceil\;\overline{3}\rceil = \overline{5 \times 3}\rceil$$

Thus the juxtaposition of operators effects multiplication in the language of the operators. We would like to remove the mark over the result $5\rceil\,3\rceil$ and thereby obtain the product of the two numbers in the realm of numbers. The following simple rule about the boundaries helps:

$$\overline{\overline{a}\rceil}\rceil = a \text{ for any a.}$$

In particular,

$$\overline{\overline{}\rceil}\rceil =$$

This rule is the Law of Crossing from Spencer-Brown's Laws of Form and has been discussed before in this column. The Law of Crossing indicates how opposite forms can fit into one another and vanish into the Void, or how the Void can produce opposite and distinct forms that fit one another, hand in glove. There is an interpretation of the Law of Crossing in terms of movement across a boundary. In this story, a mark placed over a form connotes the crossing of the boundary from the Domain indicated by that form to the Domain that is opposite to it. Thus in the double mark above, the connotation is a crossing *from* the single mark on the inside. The single mark on the inside stands for the marked state. Thus by placing a cross over it, we transit to the unmarked state. Hence the disappearance to Void on the right-hand side of the equation. The value of a crossing made again is not the value of the crossing.

We take on the Law of Crossing for this version of Spencer Brown's arithmetic (See column number 11 for the references.). With the Law of Crossing in use, we can write

$$\overline{\overline{5}\rceil\;\overline{3}\rceil}\rceil = \overline{\overline{5 \times 3}\rceil}\rceil = 5 \times 3.$$

In fact, these operators begin to take on a life of their own, but I will reserve the details of that to another place.

What I wish to focus upon here is what this formalism says about zero. Consider zero multiplied by zero. Since zero is represented by absolutely nothing, we get

$$0 \times 0 = \overline{\overline{}\rceil\;\overline{}\rceil}\rceil$$

and there is no reason in this system to reduce 0 x 0 to 0. In fact we see that

$$0 \times 0 = \overline{2}\rceil,$$

our important multiplication operator corresponding to the number two.

In the last column we considered zero numbers, non-trivial powers of zero, in just this way. Now look at the powers of zero:

$$0^0 = \overline{\;\;}|$$

$$0^1 = \overline{\overline{\;\;}|}| \quad = 0$$

$$0^2 = \overline{\overline{\;\;}|\;\overline{\;\;}|}|$$

$$0^3 = \overline{\overline{\;\;}|\;\overline{\;\;}|\;\overline{\;\;}|}|$$

Now we can see how beautifully this choice of notation works.

Zero raised to the first power is indeed zero, since

$\overline{\overline{\;\;}|}|$ is equal to absolutely nothing, and this is zero in our system.

And the most wonderful equation of all is

$$0^0 = \overline{\;\;}|$$

For here we have the production of Distinction and Unity from the Void. The Void taken to its own Power produces Unity. This equation is true. We have proven it. It is beautiful and we can not know its full meaning. For that is the Power of the Void.

Thus ends column number $\overline{\;\;}|\overline{\;\;}|\overline{\;\;}|\;\overline{\;\;}|\overline{\;\;}|\overline{\;\;}|\;\overline{\;\;}|\overline{\;\;}|\overline{\;\;}|\;\overline{\;\;}|\overline{\;\;}|\overline{\;\;}|$